**Extended Essay Outline**

It’s a plan of your extended essay, where you structure it and organize the main points into paragraphs so it would be easier for you to write an essay.

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| Student’s ID: 2022512  Student’s name: Suni Yao | Supervisor’s name: Yan Qin |
| Subject area: Mathematics Analysis and Approaches Higher Level  Title: The Extension of Marble Problem onto Analysis of Instruments in A Given Recording | |
| Research question: How Can The Marble Problem Be Extended for Analyzing The Existence of Music Instruments in A Given Record? | |

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| Outline of the extended essay:  (Please write your main idea here, prepare headings for your future paragraphs of the writing/ the detailed research methods.)   1. Introduction – Introduce my motivation for investigating this topic: it is hard to recognize which musical instruments exist in a clip of a symphony (for example, it is hard to distinguish between clarinet and saxophone). Therefore, I aim to find a solution to use algebraic method to test which instruments are persist in the clip. 2. Background: 3. Music clip: Given a music clip, by using the Fourier transformation, we can analyze the frequency and corresponding magnitude. When playing only one note of a fundamental tone *f* on a musical instrument, there exist “overtones” in frequency *nf,.* Different musical instruments have differences in playing the same note because they have different magnitudes of overtones, as shown in the Figures below. The distribution of magnitude to frequencies is different in the horn and flute. Due to the difference, we can list equations of the magnitude for every frequency. Assume the total magnitudes are *A1, A2, A3, …* For example, for instrument *a*, we can have magnitude = *a1, a2, a3, …* for every frequency *f1, f2, f3, …* for instrument *b*, we have magnitude = *b1, b2, b3, …* for these frequencies. And for instruments *c, d, e, …* they are the same. The existence of the instrument in the clip is *E(a,b,…).*       Therefore, we have *A1=E(a)a1+E(b)b1+E(c)c1+…, A2=E(a)a2+E(b)b2+E(c)c2+…* The *a1, a2, b2, c2* values are known in appliable analysis.   1. Marble problem: the marble problem is a problem that tries to get how many and which marbles are of unknown weights, which is much harder if we use the dichotomy method. In this condition, a better way is to draw some marbles randomly of several times. If we say 10 marbles and if the difference between the marble’s real weight and its standard weight is *x,* we may result in such a group of equations:   The numerical values on the right are the difference of the marbles’ weight compared to the standard weight. Due to the common sense, the bad marbles are fewer than the standard ones so we consider from the condition that there’s only one marble, only one *x* is nonzero – the 1-sparse solution. However, it would be time consuming if we first consider 1-sparse, then 2-sparse, 3, 4, … Instead, we can optimize the sparse-finding operation by this:  A here is sensing matrix or the measurement matrix, the vector b stems from the collected data. And then by processing some limiting conditions of the matrix and vector, we can find the sparse solution easily. (I will address this part more in the math draft and will hand it on before 6.20)   1. Connection: The sparse-finding procedure needs the sensing matrix and the collected data vector. Comparing the marble problem to the music-analyzing problem, the music-analyzing problem also have the aim of reducing to the least sparse (in most songs), and will contain a sensing matrix and collected data vector. The collected data vector **b** is <A1, A2, …, An>, the sensing matrix **A** is [[a1,a2,a3,…,an]; [b1, b2, b3, …, bn]; …], and **x=**<E(a), E(b), E(c), …**>.** 2. Validation: After the music-analyzing problem is solved using the sparse-finding method, I can use several samples to validate the result. First, I need to sample the corresponding magnitude of every overtone frequency of C4 note for several musical instruments, record as a1, a2, a3, …, an; b1, b2, b3, …, bn; … Then, I randomly choose some of them to play the C4 note at the same time and record the magnitude of every frequency presented, as A1, A2, …, An. Then, the result is calculated and output a string of the vector **x=<**E(a), E(b), E(c), …>, each in values of 1 and 0. Then the answer is checked to show whether the result of the research can be used effectively. (The sampling procedure can be achieved by inputting the C4 note in MuseScore and output .wav file. Program can be used for Fourier analysis and output of the frequency and amplitude. )   Limitation: I narrowed the application to where the key instruments played are totally the same, which would be an easier sparse-finding procedure. |
| Reference:  <https://www.youtube.com/watch?v=rj9NOiFLxWA&t=541s>  <https://github.com/jeffheaton/present/blob/master/youtube/video/fft-frequency.ipynb>  <https://twu-ir.tdl.org/server/api/core/bitstreams/f673965f-7aa2-41f3-ada3-392b96a489bf/content>  Bryan, Kurt, and Tanya Leise. "Making do with less: an introduction to compressed sensing." Siam Review 55.3 (2013): 547-566. <https://epubs.siam.org/doi/pdf/10.1137/110837681> |